Electromagnetics and Energy Systems Area 2011 Ph.D. Exam

**Question 1.** A uniform plane wave with monochromatic radian frequency $\omega$ is incident on a dielectric interface making an angle $\theta_i$ with respect to the normal as shown. Assume sinusoidal steady-state conditions. Both dielectric regions are lossless. Medium 1 has dielectric permittivity $\varepsilon_1$ and medium 2 has permittivity $\varepsilon_2$. Both regions have free-space permeability $\mu_0$. Assume that the incident electric filed has amplitude $E_0$ (Volts/meter).

(10 points for each part)

a) Give the analytic expressions for the incident, transmitted, and reflected wave vectors $\mathbf{k}^i$, $\mathbf{k}^t$, and $\mathbf{k}^r$, respectively.
b) Determine the velocities of propagation for the incident, reflected, and transmitted waves.
c) Write analytic expressions for the incident, transmitted, and reflected electric and magnetic fields. Your answer must include the amplitude, phase, and polarization for each field.
d) Give the analytical expression(s) that the electric and magnetic fields must satisfy at the boundary between the dielectric regions.
e) Determine the relationship among the angles $\theta_i$, $\theta_r$, and $\theta_t$.
f) What relationship among material properties ($\varepsilon_1, \varepsilon_2, \mu_0$) must exist for total internal reflection to occur?
g) Determine the condition on the angle of incidence for total internal reflection.
h) For incident angles greater than the critical angle, i.e., the angle needed for total internal reflection, determine the magnitude and the phase of the reflection coefficient $\Gamma$, where $E_r = \Gamma E_i$.
i) Determine the Brewster angle (the angle for no reflected wave to exist in medium 1) if such an angle exists for this case. If this angle does not exist state why this is so.
j) Qualitatively describe what would change if medium 2 were slightly lossy. Would the wave in region 2 still propagate as a uniform plane wave?
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**Question 2.** Two infinitely long parallel wires are suspended by massless strings and are aligned parallel to the y-direction. Each wire carries equal current of magnitude \( I \).

(25 points for each part)

a) Are the currents co-propagating or counter-propagating?
b) For an arbitrary value of current \( I \) flowing in either the \( \pm y \)-direction on each wire, write the expression for the magnetic force exerted on each wire. Give both the magnitude and direction of the force.
c) If the wire itself weighs 0.2 grams/meter, and if the angle between the wires is 10° (as shown in the figure), determine the magnitude of the current that flows on each wire. The acceleration of gravity is 9.8 m/sec² and \( \mu_0 = 4\pi \times 10^{-7} \)
d) What current would be required for the angular separation to equal 90°?