EM

Ph.D. Qualifying Exam – Spring 2015

Electromagnetics and Energy Systems Area

Table of Useful Taylor Series Expansions for Small x:

$$
\begin{aligned}
(1+x)^n &\approx 1+nx \\
e^x &\approx 1+x \\
a^x &\approx 1+x\ln a \\
\ln(1+x) &\approx x \\
\sin x &\approx x \\
\cos x &\approx 1 \\
\tan x &\approx x
\end{aligned}
$$

Table of Useful Integrals:

$$
\begin{aligned}
\int \frac{dx}{(a+bx+cx^2)^{3/2}} &= \frac{4cx+2b}{(4ac-b^2)\sqrt{a+bx+cx^2}} + \text{const.} \\
\int \frac{dx}{x^2+b^2} &= \frac{1}{b}\arctan\left(\frac{x}{b}\right) + \text{const.} \\
\int \frac{xdx}{x^2+b^2} &= \frac{1}{2}\ln\left(x^2+b^2\right) + \text{const.} \\
\int \frac{dx}{(a+bx+cx^2)^{1/2}} &= \frac{1}{\sqrt{c}}\ln\left(2\sqrt{ac+bcx+c^2x^2}+2cx+b\right) + \text{const.} \quad [c>0] \\
\int a\sin(bx)dx &= -\frac{a}{b}\cos(bx) + \text{const.} \\
\int a\cos(bx)dx &= \frac{a}{b}\sin(bx) + \text{const.} \\
\int_0^r \frac{a}{x}\sin(bx)dx &= a\text{Si}(br) \\
\int_0^r \frac{a}{x}\cos(bx)dx &= a\text{Ci}(br) \\
\int ax\sin(bx)dx &= -\frac{ax}{b}\cos(bx) + \frac{a}{b^2}\sin(bx) + \text{const.} \\
\int ax\cos(bx)dx &= \frac{ax}{b}\sin(bx) + \frac{a}{b^2}\cos(bx) + \text{const.}
\end{aligned}
$$

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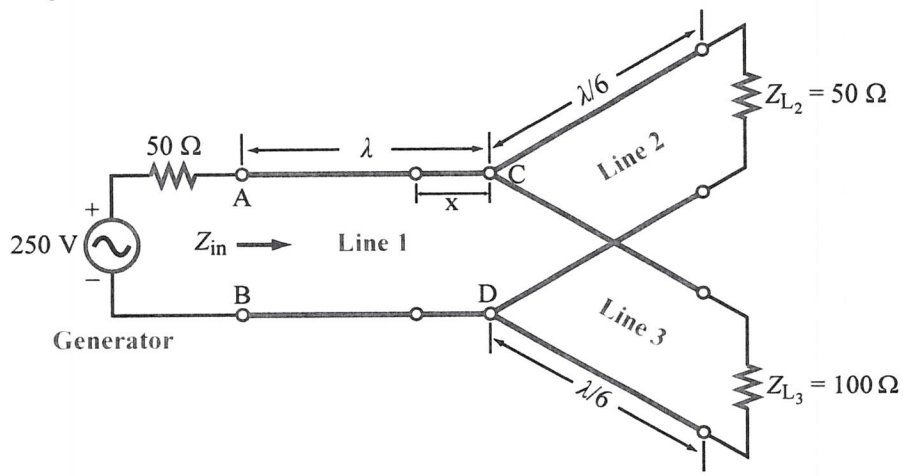
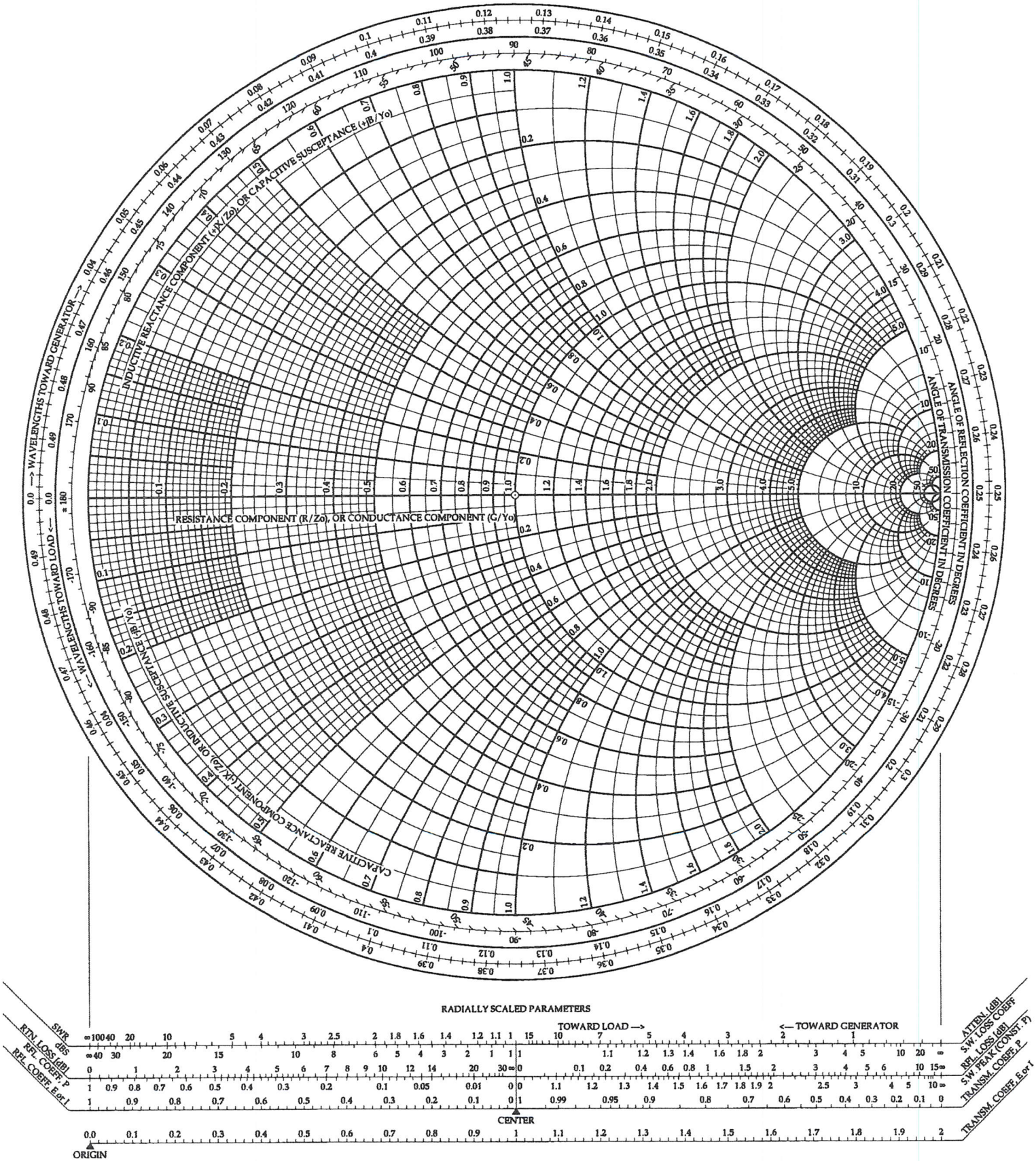
1. **Transmission Line Matching.** For the transmission line configuration shown below, an engineer wishes to insert a shunt capacitor at a distance x from the $C-D$ junction to match the effective impedance of the combined loads. Assume the characteristic impedances of the lines are $Z_1 = Z_2 = Z_3 = 50\ \Omega$, and that the phase velocity, v_p, for each line is equal to the speed of light, c. Smith Charts are provided on the following pages, for your convenience.

   (a) (50%) Determine the effective impedance at junction $C-D$ that results from the two load resistances, Z_{L2} and Z_{L3}.

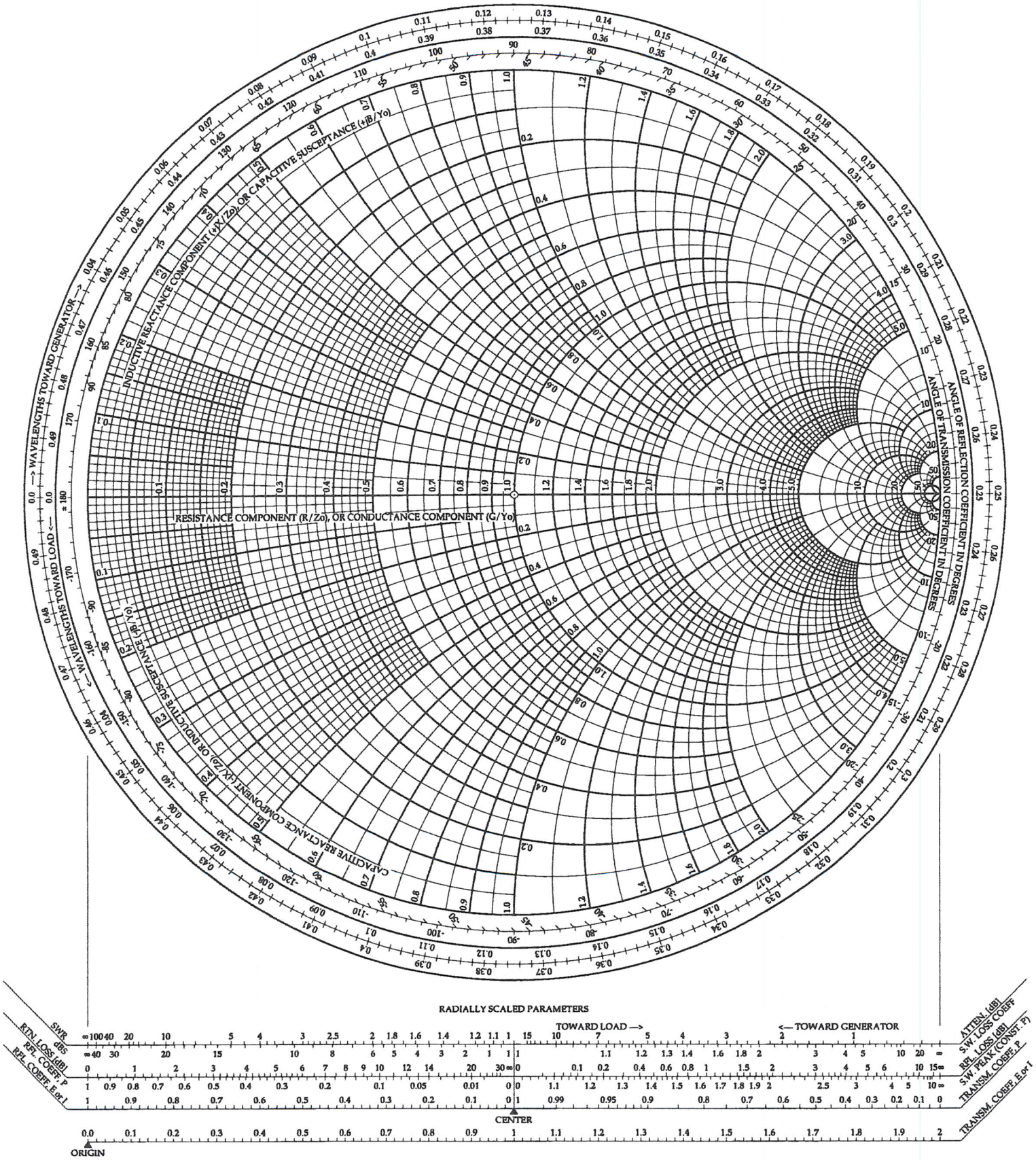
   (b) (50%) Determine the distance, x, and the capacitance, C, of the engineer's shunt capacitor, assuming an operating frequency of 300 kHz. You should express the distance in units of wavelength, λ, and the capacitance in Farads.



EM1


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
SWR
RTN. LOSS (dB)
REFL. COEFF. P
REFL. COEFF. E or I
ATTEN. (dB)
S.W. LOSS COEFF
REFL. LOSS (dB)
S.W. PEAK CONST. F)
TRANSM. COEFF. P
TRANSM. COEFF. E or I
CENTER
ORIGIN

EM1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SWR
RTN. LOSS [dB]
REFL. COEFF. P
REFL. COEFF. E or I
ATTEN. [dB]
S.W. LOSS COEFF
REFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF. E or I
TRANSM. COEFF. P
CENTER
ORIGIN

EM2

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2. Sphere of charge. The electric field produced by a sphere of charge of radius a with density $\rho(r)$ is given by:

$$E_r = \begin{cases} r^3 + Ar^2 & r \leq a \\ \left(a^5 + Aa^4\right)/r^2 & r \geq a \end{cases}$$

(100%) Determine the charge distribution $\rho(r)$ assuming it only varies with r.