SIGNALS AND SYSTEMS QUALIFYING EXAMINATION TABLES

You may find the following tables useful:
From B. P. Lathi, *Linear Systems and Signal*, Oxford University Press, 2nd ed., 2005.

TABLE 7.1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	

From J. J. Komo, *Random Signal Analysis in Engineering Systems*, Academic Press, 1987.

$$e^{jx} = \cos x + j\sin x$$

$$\cos x = \frac{e^{jx}+e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\cos\left(x\pm\frac{\pi}{2}\right) = \mp\sin x$$

$$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\sin\left(x\pm\frac{\pi}{2}\right) = \pm\cos x$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos x+\cos y = 2\cos[\tfrac{1}{2}(x+y)]\cos[\tfrac{1}{2}(x-y)]$$

$$\cos x-\cos y = -2\sin[\tfrac{1}{2}(x+y)]\sin[\tfrac{1}{2}(x-y)]$$

$$\sin x\pm\sin y = 2\sin\tfrac{1}{2}(x\pm y)\cos\tfrac{1}{2}(x\mp y)$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos^2 x = \tfrac{1}{2}(1+\cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1-\cos 2x)$$

$$A\cos x - B\sin x = R\cos(x+\theta)$$

where $R=\sqrt{A^2+B^2}$ and $\theta=\tan^{-1}\left(\dfrac{B}{A}\right)$

SIGNALS AND SYSTEMS QUALIFYING EXAMINATION
Question 1

Consider a discrete-time linear time-invariant system whose transfer function has two poles at $z = 0.8$ and $z = -0.5$, and one zero of order 2 at $z = 0$.

(a) *(10 points)* Is the system bounded-input bounded-output (BIBO) stable? Very briefly justify your answer.

(b) *(50 points)* Find the impulse response $h[n]$ of the system. Show your work. <u>Hint:</u> You may need the z-transfrom relation $c^n u[n] \longleftrightarrow \frac{1}{1-cz^{-1}}$, where $u[n]$ is the unit step signal.

(c) *(40 points)* Let $x[n] = 10(0.8)^n u[n]$, where $u[n]$ is the unit step signal. Find the output of the system when the input is $x[n]$. Show your work.

SIGNALS AND SYSTEMS QUALIFYING EXAMINATION
Question 2

(a) *(40 points)* Let a_k be the kth Fourier series coefficient of a periodic signal $x(t)$ whose fundamental period is T_0. Let b_k be the kth Fourier series coefficient of the delayed signal $x(t - T_0/2)$. Show that $b_k = (-1)^k a_k$.

(b) *(60 points)* Let the continuous-time signal $g(t)$ be band-limited to W Hz, i.e., its Fourier transform $G(f) = 0$ for $|f| > W$. Consider the signal $y(t) = 2g(t)\cos^2(20\pi W t)$. Determine the Fourier transform $Y(f)$ of $y(t)$. Show your work.