TABLE 7.1 Fourier Transforms

<table>
<thead>
<tr>
<th>No.</th>
<th>$x(t)$</th>
<th>$X(\omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$e^{-at}u(t)$</td>
<td>$\frac{1}{a + j\omega}$</td>
</tr>
<tr>
<td>2</td>
<td>$e^{at}u(-t)$</td>
<td>$\frac{1}{a - j\omega}$</td>
</tr>
<tr>
<td>3</td>
<td>$e^{-</td>
<td>t</td>
</tr>
<tr>
<td>4</td>
<td>$te^{-at}u(t)$</td>
<td>$\frac{1}{(a + j\omega)^2}$</td>
</tr>
<tr>
<td>5</td>
<td>$t^ne^{-at}u(t)$</td>
<td>$\frac{n!}{(a + j\omega)^{n+1}}$</td>
</tr>
<tr>
<td>6</td>
<td>$\delta(t)$</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>$2\pi \delta(\omega)$</td>
</tr>
<tr>
<td>8</td>
<td>$e^{j\omega_0 t}$</td>
<td>$2\pi \delta(\omega - \omega_0)$</td>
</tr>
<tr>
<td>9</td>
<td>$\cos \omega_0 t$</td>
<td>$\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$</td>
</tr>
<tr>
<td>10</td>
<td>$\sin \omega_0 t$</td>
<td>$j\pi [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$</td>
</tr>
</tbody>
</table>


$$e^{jx} = \cos x + j \sin x$$
$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\cos\left(\frac{x \pm \pi}{2}\right) = \mp \sin x$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\sin\left(\frac{x \pm \pi}{2}\right) = \pm \cos x$$
$$\cos x \cos y = \frac{1}{2} \left[ \cos(x + y) + \cos(x - y) \right]$$

$$\sin x \sin y = \frac{1}{2} \left[ \cos(x - y) - \cos(x + y) \right]$$
$$\sin x \cos y = \frac{1}{2} \left[ \sin(x + y) + \sin(x - y) \right]$$
$$\cos x + \cos y = 2 \cos\left[ \frac{1}{2}(x + y) \right] \cos\left[ \frac{1}{2}(x - y) \right]$$
$$\cos x - \cos y = -2 \sin\left[ \frac{1}{2}(x + y) \right] \sin\left[ \frac{1}{2}(x - y) \right]$$
$$\sin x \pm \sin y = 2 \sin \frac{1}{2}(x \pm y) \cos \frac{1}{2}(x \mp y)$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$
$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$
$$A \cos x - B \sin x = R \cos(x + \theta)$$

where $R = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1}\left(\frac{B}{A}\right)$
SIGNALS AND SYSTEMS QUALIFYING EXAMINATION

Question 1

Consider a discrete-time linear time-invariant system whose transfer function has two poles at $z = 0.8$ and $z = -0.5$, and one zero of order 2 at $z = 0$.

(a) (10 points) Is the system bounded-input bounded-output (BIBO) stable? Very briefly justify your answer.

(b) (50 points) Find the impulse response $h[n]$ of the system. Show your work. Hint: You may need the $z$-transform relation $e^n u[n] \longleftrightarrow \frac{1}{1-e^{-z}}$, where $u[n]$ is the unit step signal.

(c) (40 points) Let $x[n] = 10(0.8)^n u[n]$, where $u[n]$ is the unit step signal. Find the output of the system when the input is $x[n]$. Show your work.
SIGNALS AND SYSTEMS QUALIFYING EXAMINATION

Question 2

(a) (40 points) Let $a_k$ be the $k$th Fourier series coefficient of a periodic signal $x(t)$ whose fundamental period is $T_0$. Let $b_k$ be the $k$th Fourier series coefficient of the delayed signal $x(t - T_0/2)$. Show that $b_k = (-1)^k a_k$.

(b) (60 points) Let the continuous-time signal $g(t)$ be band-limited to $W$ Hz, i.e., its Fourier transform $G(f) = 0$ for $|f| > W$. Consider the signal $y(t) = 2g(t) \cos^2(20\pi W t)$. Determine the Fourier transform $Y(f)$ of $y(t)$. Show your work.