Question 1) Basic physics of semiconductors and pn junctions

Consider an ideal $p^+\text{-}n$ junction (i.e. $N_a >> N_d$) at 300 K, with a cross-sectional area $A = 10^{-3}$ cm$^2$.

For a $p-n$ junction, the $I$-$V$ relationship is given by $I = qA \left( \frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) \left( e^{qV/kT} - 1 \right)$.

For a $p-n$ junction, the depletion region width is given by $W = \sqrt{\frac{2 \varepsilon_0 \varepsilon_r (V_0 - V)}{q \left( \frac{N_a + N_d}{N_a N_d} \right)}}$.

a) Suppose that the built-in (contact) potential is $V_0 = 0.5$ V and the donor concentration on the $n$-side is $N_d = 4 \times 10^{15}$ cm$^{-3}$. Assume that the depletion region width $W$ is in the $n$-side of the junction entirely and calculate $W$ at equilibrium. (30 points).

b) If the hole diffusion coefficient is $D_p = 10$ cm$^2$/s and the hole diffusion length is $L_p = 3.16 \times 10^{-3}$ cm, calculate the current at a forward bias of 0.7 V at 300 K. (30 points).

c) Now assume that $D_a = D_p$ and $L_n = L_p$ and the diode is forward biased. What fraction of the forward diffusion current is due to minority carrier injection of electrons into the $p^+$ region if the acceptor concentration on the $p$-side is $N_a = 10^{19}$ cm$^{-3}$? (20 points).

d) Now assume that we fabricate an identical Germanium (Ge) $p^+\text{-}n$ junction instead of Silicon (Si). Assume for this problem that all materials properties of Si and Ge are identical except that Ge has a smaller bandgap than Si. Will the Si or Ge $p^+\text{-}n$ junction have a higher reverse saturation current $I_0$? Explain in a few sentences using appropriate equations to justify your answer. (20 points).

Physical constants:
$q = 1.6 \times 10^{-19}$ C
$kT$ (at $T=300$ K) = 0.0259 eV
$\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm

Properties of Silicon:
$E_g = 1.11$ eV
$\alpha = 11.8$
$n_i$ (at $T=300$ K) = $1.5 \times 10^{10}$ cm$^{-3}$
Question 2) Quantum Mechanics

Consider a 1D electron wavefunction given by

\[ \psi_n(x,t) = A \sin\left(\frac{n\pi x}{L}\right) \exp\left(-\frac{iEt}{\hbar}\right) \]

where \( A \) is the normalization constant, \( L \) and \( E \) are also constants, and \( n = 1,2,3,4,\ldots \) (i.e. a positive integer).

Assume that the wavefunction is zero (i.e. \( \psi_n(x,t) = 0 \)) when \( x < 0 \) and \( x > L \).

a) Calculate the value of the normalization constant \( A \). (30 points).

b) Calculate the probability of finding the electron in the region \( 0 \leq x \leq L/4 \) for arbitrary \( n \). (30 points).

c) What is the probability of finding the electron in the region \( 0 \leq x \leq L/2 \) for arbitrary \( n \)? For this part, give the answer without calculating any integrals. (10 points).

d) Calculate the expectation value of the momentum of the electron for arbitrary \( n \). (30 points).

Potentially useful integral formulas:

\[ \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad \int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \]

\[ \int x \cos^2 ax \, dx = \frac{x^2}{4} + \frac{x \sin 2ax}{4a} + \frac{\cos 2ax}{8a^2} \]

\[ \int x \sin^2 ax \, dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2} \]

\[ \int \sin ax \cos ax \, dx = \frac{\sin^2 ax}{2a} \]